MODIFIED PARTIAL UPDATE EDS ALGORITHMS FOR ADAPTIVE FILTERING

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ABSTRACT

Partial update (PU) Euclidean direction search (EDS) algorithms have been developed to reduce the computational complexity of the full-update EDS. In this paper, the PU EDS is modified to achieve better performance. The performance is analyzed for a time-invariant system and for a time-varying system. Theoretical steady-state mean and MSE results of the modified PU EDS are derived for both time-invariant system and time-varying system. Computer simulations are presented to support the theoretical analyses. The modified PU EDS can achieve similar performance to the full update EDS while reducing the computational complexity significantly. The performance of the modified PU EDS is also compared with the PU recursive least squares (RLS) algorithm and the PU conjugate gradient (CG) in computer simulations. The performance of modified PU EDS is comparable to PU RLS, and it needs less computational cost.

1. INTRODUCTION

Adaptive filters play an important role in fields related to digital signal processing such as system identification, noise cancellation, and channel equalization. In the real world, the computational complexity of an adaptive filter is an important consideration for applications which need long filters. Usually, least squares algorithms, such as recursive least squares (RLS), Euclidean direction search (EDS) [1], and conjugate gradient (CG), have higher computational complexity and give better convergence performance than steepest-descent algorithms. Therefore, a tradeoff must be made between computational complexity and performance. To reduce the computational complexity, one option is to use partial update techniques [2]. The partial update adaptive filter only updates part of the coefficient vector instead of updating the entire vector. The theoretical results of the full-update case may not apply to the partial update case. Therefore, performance analysis of the partial update adaptive filter is very meaningful. In the literature, there are few studies for partial update least squares algorithms. In [3], the mean and mean-square performance of the MMax RLS has been analyzed for white inputs. In [4], the tracking performance has been analyzed for MMax RLS. In [5], partial update techniques have been applied to the CG algorithm. The mean and mean-square performance of different PU CG algorithms were analyzed in a time-invariant system. In [6], the tracking performance of MMax CG was analyzed. In [8], the mean and mean-square performance of PU EDS were studied. However, the MMax, sequential, and stochastic EDS do not converge to the same steady-state mean-square-error (MSE) as the full update EDS.

In this paper, the PU EDS is modified to achieve better performance. Theoretical steady-state mean and MSE results of the modified PU EDS are derived for both the time-invariant system and time-varying system. Computer simulation results are also presented to show the performance of the modified PU EDS. The performance of the MMax EDS is also compared with the full-update EDS, full-update RLS, MMax RLS, CG, and MMax CG. The analysis of time-varying systems is necessary because the unknown systems in system identification, echo cancellation, and channel equalization are often time-varying in real world applications. This paper is organized as follows. In Section 2, the modified PU EDS algorithms are derived. The mean and MSE results of the modified PU EDS for a time-invariant system are derived in Section 3. The MSE results of the PU EDS for a time-varying system are derived in Section 4. In Section 5, computer simulation results are shown.

2. PARTIAL UPDATE EDS

The partial update EDS is briefly reviewed in this section. A system identification model is shown in Figure 1. It can be written as

$$d(n) = \mathbf{x}^T(n)\mathbf{w}^o + v(n), \tag{1}$$

where $d(n)$ is the desired signal, $\mathbf{x}(n) = [x(n), x(n-1), ..., x(n-N+1)]^T$ is the input data vector of the unknown system, $\mathbf{w}^o = [w_1^o, w_2^o, ..., w_N^o]^T$ is the impulse response vector of the unknown system, and $v(n)$ is zero-mean white

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noise, which is independent of any other signal. In a stationary environment, $\mathbf{w}^o$ is time-invariant. In a non-stationary environment, $\mathbf{w}^o$ is time-varying.

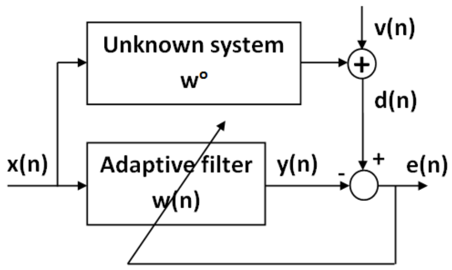

Fig. 1. System identification model.

Let $\mathbf{w}$ be the coefficient vector of an adaptive filter. The estimated signal $y(n)$ is defined as

$$y(n) = \mathbf{x}^T(n)\mathbf{w}(n-1), \tag{2}$$

and the output signal error is defined as

$$e(n) = d(n) - \mathbf{x}^T(n)\mathbf{w}(n-1). \tag{3}$$

The EDS algorithm solves the same least-squares cost function as the RLS and CG. It aims to minimize the cost function

$$J(n) = \frac{1}{2}\mathbf{w}^T(n)\mathbf{Q}\mathbf{w}(n) - \mathbf{r}^T\mathbf{w}(n), \tag{4}$$

where $\mathbf{Q} = E\{\mathbf{x}(n)\mathbf{x}^T(n)\}$ is the autocorrelation matrix of the input data vector $\mathbf{x}(n)$ and $\mathbf{r} = E\{\mathbf{x}(n)d(n)\}$ is the cross-correlation vector between the input data vector $\mathbf{x}(n)$ and the desired signal $d(n)$. Unlike RLS, the EDS minimizes the cost function using the simplest direction search method – Euclidean direction search [1] to avoid matrix inversion and reduce the computational complexity. The partial update method aims to reduce the computational cost of the adaptive filters. Instead of updating all of the $N \times 1$ coefficients, it usually only updates $M \times 1$ coefficients, where $M < N$. EDS algorithm uses $\mathbf{Q}(n)$ and $\mathbf{r}(n)$ to estimate the autocorrelation matrix $\mathbf{Q}$ and crosscorrelation vector $\mathbf{r}$. Calculation of $\mathbf{Q}(n)$ results in high computational cost. To reduce the computational complexity, the partial update estimated autocorrelation matrix $\widetilde{\mathbf{Q}}(n)$ is used.

The modified partial update EDS has the uniform update equation

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mathbf{g}\mathbf{g}^T(\widetilde{\mathbf{Q}}(n)\mathbf{w}(n) - \hat{\mathbf{r}}(n))}{\mathbf{g}^T\widetilde{\mathbf{Q}}(n)\mathbf{g}}, \tag{5}$$

where

$$\widetilde{\mathbf{Q}}(n) = \sum_{i=1}^{n} \lambda^{n-i}\mathbf{x}(i)\hat{\mathbf{x}}^T(i), \tag{6}$$

$$\hat{\mathbf{r}}(n) = \sum_{i=1}^{n} \lambda^{n-i}\hat{\mathbf{x}}(i)d(i), \tag{7}$$

$$\hat{\mathbf{x}} = \mathbf{I}_M\mathbf{x}, \tag{8}$$

$$\mathbf{I}_M(n) = \begin{bmatrix} i_1(n) & 0 & \dots & 0 \\ 0 & i_2(n) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & i_N(n) \end{bmatrix}, \tag{9}$$

$$\sum_{k=1}^{N} i_k(n) = M, \quad i_k(n) \in \{0,1\}, \tag{10}$$

the $N \times 1$ vector $\mathbf{g}$ is the search direction at iteration n, which is taken to be "Euclidean directions." It is defined as $\mathbf{g}_i = [0, \cdots, 0, 1, 0, \cdots, 0]^T$, where the 1 appears in the i-th position. At each iteration n, the entire weight vector $\mathbf{w}(n)$ is updated by cycling through all the Euclidean directions $\mathbf{g}_i$, $i = 1, 2, \cdots, N$ [7]. The forgetting factor $0 < \lambda < 1$ gives exponentially less weight to previous samples. Unlike the PU EDS in [8], the PU estimated autocorrelation matrix is modified to $\widetilde{\mathbf{Q}}(n) = \sum_{i=1}^{n} \lambda^{n-i}\mathbf{x}(i)\hat{\mathbf{x}}^T(i)$ instead of $\hat{\mathbf{Q}}(n) = \sum_{i=1}^{n} \lambda^{n-i}\hat{\mathbf{x}}(i)\hat{\mathbf{x}}^T(i)$. Although the modified PU EDS requires a slight bit more in computational costs than the original PU EDS, the performance is improved significantly. The modified PU EDS can achieve similar steady-state MSE to the full update EDS. Note, the calculation of output signal error still uses the the whole input vector, not the subselected input vector. Basic partial update methods including sequential PU, stochastic PU, and MMax method are applied to EDS. The sequential PU method chooses the input vector subsets in a round-robin fashion. The stochastic PU method chooses the input vector subsets randomly. Usually a uniformly distributed random process will be applied. The MMax EDS selects the input vector according to the M greatest entries of the input vector $\mathbf{x}$ in absolute value. The sorting of the input $\mathbf{x}$ increases the computational complexity. The sorting result can be achieved more efficiently by using the SORTLINE or Short-sort methods [9]. If the SORTLINE method is used, the MMax EDS needs $2 + 2\lceil log_2 N \rceil$ comparisons. The total number of multiplications needed for the uniform PU EDS is $N^2 + 2NM + N + 2M$. The original EDS needs $3N^2 + 3N$ [1] multiplications. If M is much smaller than N, then the number of multiplications can be reduced significantly for PU EDS.

3. PERFORMANCE OF MODIFIED PARTIAL UPDATE EDS FOR TIME-INVARIANT SYSTEM

The mean behavior of the modified PU EDS weights can be determined by multiplying a scalar $\mathbf{g}^T\widetilde{\mathbf{Q}}(n)\mathbf{g}$ to both sides of (5) and taking the expectation

$$\begin{aligned} E\{\mathbf{g}^T\widetilde{\mathbf{Q}}(n)\mathbf{g}\mathbf{w}(n+1)\} &= E\{\mathbf{g}^T\widetilde{\mathbf{Q}}(n)\mathbf{g}\mathbf{w}(n)\} \\ - \quad E\{\mathbf{g}\mathbf{g}^T(\widetilde{\mathbf{Q}}(n)\mathbf{w}(n) &- \hat{\mathbf{r}}(n))\}. \end{aligned} \tag{11}$$

Assume $\widetilde{\mathbf{Q}}(n)$ and $\mathbf{w}(n)$ are uncorrelated to each other. $\mathbf{g}\mathbf{g}^T$ is just direction and is uncorrelated to $\widetilde{\mathbf{Q}}(n)$ and $\hat{\mathbf{r}}(n)$. At steady state, $E\{\mathbf{g}^T\widetilde{\mathbf{Q}}(n)\mathbf{g}\mathbf{w}(n+1)\} = E\{\mathbf{g}^T\widetilde{\mathbf{Q}}(n)\mathbf{g}\mathbf{w}(n)\}$. Therefore, (11) can be simplified to

$$E\{\mathbf{g}\mathbf{g}^T\}E\{\widetilde{\mathbf{Q}}(n)\}E\{\mathbf{w}(n)\} = E\{\mathbf{g}\mathbf{g}^T\}E\{\hat{\mathbf{r}}(n)\}. \tag{12}$$

At steady state, $E\{\widetilde{\mathbf{Q}}(n)\} = \frac{1}{1-\lambda}\widetilde{\mathbf{Q}}$ and $E\{\hat{\mathbf{r}}(n)\} = \frac{1}{1-\lambda}\hat{\mathbf{r}}$, where $\widetilde{\mathbf{Q}} = E\{\mathbf{x}(n)\hat{\mathbf{x}}^T(n)\}$ and $\hat{\mathbf{r}} = E\{\hat{\mathbf{x}}(n)d(n)\}$. If the inversion of $\widetilde{\mathbf{Q}}$ exists, the mean weights of the modified PU EDS converge to

$$E\{\mathbf{w}(n)\} = \widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{r}} \qquad n \to \infty. \tag{13}$$

For the MMax method $\widetilde{\mathbf{Q}}$ is close to that of the full update method. For the sequential and stochastic methods, $\widetilde{\mathbf{Q}}$ is different from that of the full update method. The inversion of $\widetilde{\mathbf{Q}}$ does not always exist, especially when the PU length is small.

The coefficient error vector is defined as

$$\mathbf{z}(n) = \mathbf{w}(n) - \mathbf{w}^o. \tag{14}$$

To derive the MSE performance at steady state, three assumptions are needed: (1) Inversion of $\widetilde{\mathbf{Q}}$ exists; (2) at steady state, the coefficient error vector $\mathbf{z}(n)$ is very small and is independent of the input signal $\mathbf{x}(n)$; (3) the input signal $\mathbf{x}(n)$ is independent of noise $v(n)$.

Define the weight error correlation matrix as

$$\mathbf{K}(n) \quad = \quad E\{\mathbf{z}(n)\mathbf{z}^T(n)\}. \tag{15}$$

Using the assumptions, the MSE equation of the PU EDS algorithm at steady state becomes

$$E\{|e(n)|^2\} = \sigma_v^2 + tr(\mathbf{Q}\mathbf{K}(n)), \tag{16}$$

where $\mathbf{Q}$ is the autocorrelation matrix of the input $\mathbf{x}$.

At steady state, the coefficient vector is approximate to

$$\mathbf{w}(n) \approx \widetilde{\mathbf{Q}}^{-1}(n)\hat{\mathbf{r}}(n). \tag{17}$$

Assuming a slow adaptive process (λ is very close to unity), $\widetilde{\mathbf{Q}}(n)$ becomes [10]

$$\widetilde{\mathbf{Q}}(n) \approx \frac{\widetilde{\mathbf{Q}}}{1-\lambda} \qquad n \to \infty. \tag{18}$$

The coefficient vector is further approximated to

$$\begin{aligned} \mathbf{w}(n) &\approx (1-\lambda)\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{r}}(n) \\ &= \lambda\mathbf{w}(n-1) + (1-\lambda)\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\mathbf{x}^T(n)\mathbf{w}^o \\ &+ (1-\lambda)\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\mathbf{v}(n). \end{aligned} \tag{19}$$

Subtracting $\mathbf{w}^o$ from both sides of (19), using (14) and the direct-averaging method [10], we get

$$\mathbf{z}(n) \approx \lambda\mathbf{z}(n-1) + (1-\lambda)\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\mathbf{v}(n). \tag{20}$$

Note, the term $(1-\lambda)\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\mathbf{x}^T(n)\mathbf{w}^o$ in (19) becomes $(1-\lambda)\widetilde{\mathbf{Q}}^{-1}E\{\hat{\mathbf{x}}(n)\mathbf{x}^T(n)\}\mathbf{w}^o = (1-\lambda)\mathbf{w}^o$ after using the direct-averaging method.

Since the input noise is assumed to be white,

$$E\{v(i)v(j)\} = \begin{cases} \sigma_v^2 & \text{for } i = j \\ 0 & \text{otherwise} \end{cases}. \tag{21}$$

$\mathbf{K}(n)$ becomes

$$\begin{aligned} \mathbf{K}(n) &\approx \lambda^2\mathbf{K}(n-1) \\ &+ \sigma_v^2(1-\lambda)^2E\{\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)\widetilde{\mathbf{Q}}^{-T}\} \end{aligned} \tag{22}$$

At steady state $\mathbf{K}(n) \approx \mathbf{K}(n-1)$, therefore $\mathbf{K}(n)$ becomes

$$\mathbf{K}(n) \quad \approx \quad \frac{1-\lambda}{1+\lambda}\sigma_v^2\widetilde{\mathbf{Q}}^{-1}E\{\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)\}\widetilde{\mathbf{Q}}^{-T}. \tag{23}$$

The MSE equation becomes

$$E\{|e(n)|^2\} \approx \sigma_v^2 + tr(\mathbf{Q}(\frac{1-\lambda}{1+\lambda}\sigma_v^2\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{Q}}\widetilde{\mathbf{Q}}^{-T})), \tag{24}$$

where $tr(\cdot)$ is the trace operator and $\hat{\mathbf{Q}} = E\{\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)\}$.

For a white input signal with variance σ_x^2, the MSE can be simplified as

$$E\{|e(n)|^2\} \quad \approx \quad \sigma_v^2 + \frac{N(1-\lambda)}{1+\lambda}\sigma_v^2\sigma_x^2\sigma_{\hat{x}}^2\sigma_{\tilde{x}}^{-4}, \tag{25}$$

where $\sigma_{\hat{x}}^2\mathbf{I} = E\{\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)\}$ and $\sigma_{\tilde{x}}^{-2}\mathbf{I} = \widetilde{\mathbf{Q}}^{-1}$.

For the PU method and a white input signal, $\sigma_{\hat{x}}^2 \approx \kappa\sigma_x^2$ and $\sigma_{\tilde{x}}^2 \approx \kappa\sigma_x^2$, where κ is smaller than 1 and is close to 1. Therefore, the MSE can be further simplified as

$$E\{|e(n)|^2\} \approx \sigma_v^2 + \frac{N(1-\lambda)}{(1+\lambda)\kappa}\sigma_v^2. \tag{26}$$

4. PERFORMANCE OF MODIFIED PARTIAL UPDATE EDS FOR TIME-VARYING SYSTEM

In a non-stationary environment, the unknown system is time-varying. The desired signal can be rewritten as

$$d(n) = \mathbf{x}^T(n)\mathbf{w}^o(n) + v(n). \tag{27}$$

A first-order Markov model [10] is used for the time-varying unknown system. It has the form

$$\mathbf{w}^o(n) = \gamma \mathbf{w}^o(n-1) + \eta(n), \tag{28}$$

where γ is a fixed parameter of the model and is assumed to be very close to unity. $\eta(n)$ is the process noise vector with zero mean and correlation matrix $\mathbf{Q}_\eta$.

The coefficient error vector for the time-varying system is defined as

$$\mathbf{z}(n) = \mathbf{w}(n) - \mathbf{w}^o(n). \tag{29}$$

To determine the tracking performance of the modified partial update EDS, two more assumptions are needed [10]: (1) Noise $v(n)$ has zero mean and variance σ_v^2, and is independent of the noise $\eta(n)$; (2) the input signal $\mathbf{x}(n)$ is independent of both noise $v(n)$ and noise $\eta(n)$.

Using the same derivation steps as the time-invariant system, the coefficient vector at steady state is approximated to

$$\begin{aligned} \mathbf{w}(n) \approx\ & \lambda \mathbf{w}(n-1) + (1-\lambda)\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\mathbf{x}^T(n)\mathbf{w}^o(n) \\ +\ & (1-\lambda)\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\mathbf{v}(n). \end{aligned} \tag{30}$$

Subtracting $\mathbf{w}^o(n)$ from both sides of (30), using (28) and (29), using the direct-averaging method [10], and applying the assumption that γ in (28) is very close to unity, we obtain

$$\mathbf{z}(n) \approx \lambda \mathbf{z}(n-1) - \lambda \eta(n) + (1-\lambda)\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\mathbf{v}(n). \tag{31}$$

$\mathbf{K}(n)$ becomes

$$\begin{aligned} \mathbf{K}(n) \approx\ & \lambda^2 \mathbf{K}(n-1) + \lambda^2 \mathbf{Q}_\eta \\ +\ & \sigma_v^2 (1-\lambda)^2 E\{\widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)\widetilde{\mathbf{Q}}^{-T}\} \end{aligned} \tag{32}$$

At steady state $\mathbf{K}(n) \approx \mathbf{K}(n-1)$, therefore $\mathbf{K}(n)$ becomes

$$\begin{aligned} \mathbf{K}(n) \approx\ & \frac{1-\lambda}{1+\lambda}\sigma_v^2 \widetilde{\mathbf{Q}}^{-1} E\{\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)\}\widetilde{\mathbf{Q}}^{-T} \\ +\ & \frac{\lambda^2}{1-\lambda^2}\mathbf{Q}_\eta. \end{aligned} \tag{33}$$

The MSE equation becomes

$$\begin{aligned} E\{|e(n)|^2\} \approx\ & \sigma_v^2 + tr(\mathbf{Q}(\frac{1-\lambda}{1+\lambda}\sigma_v^2 \widetilde{\mathbf{Q}}^{-1}\hat{\mathbf{Q}}\widetilde{\mathbf{Q}}^{-T} \\ +\ & \frac{\lambda^2}{1-\lambda^2}\mathbf{Q}_\eta)). \end{aligned} \tag{34}$$

For a white input signal with variance σ_x^2, the MSE can be simplified as

$$\begin{aligned} E\{|e(n)|^2\} \approx\ & \sigma_v^2 + \frac{N(1-\lambda)}{1+\lambda}\sigma_v^2\sigma_x^2\sigma_{\hat{x}}^2\sigma_{\widetilde{x}}^{-4} \\ +\ & \frac{\lambda^2}{1-\lambda^2}\sigma_x^2 tr(\mathbf{Q}_\eta). \end{aligned} \tag{35}$$

For the PU method and a white input signal, the MSE can be further simplified as

$$E\{|e(n)|^2\} \approx \sigma_v^2 + \frac{N(1-\lambda)}{(1+\lambda)\kappa}\sigma_v^2 + \frac{\lambda^2}{1-\lambda^2}\sigma_x^2 tr(\mathbf{Q}_\eta). \tag{36}$$

Assume the process noise is white with variance σ_η^2. Then, the MSE of the modified PU EDS can be further simplified as

$$E\{|e(n)|^2\} \approx \sigma_v^2 + \frac{N(1-\lambda)}{(1+\lambda)\kappa}\sigma_v^2 + \frac{N\lambda^2}{1-\lambda^2}\sigma_x^2\sigma_\eta^2. \tag{37}$$

5. SIMULATIONS

5.1. Performance of the modified PU EDS for a time-invariant system

The system identification model is shown in Figure 1. It is a 16-order FIR filter (N=16). The impulse response [11] is

$$\begin{aligned} \mathbf{w}^o = [&0.01, 0.02, -0.04, -0.08, 0.15, -0.3, 0.45, 0.6, \\ &0.6, 0.45, -0.3, 0.15, -0.08, -0.04, 0.02, 0.01]^T. \end{aligned} \tag{38}$$

In our simulations, the lengths of the partial update filter are M=8 and M=4. The variance of the input noise $v(n)$ is 0.0001. The initial weights of the EDS are $\mathbf{w} = \mathbf{0}$, the initial autocorrelation matrix $\mathbf{Q}(0) = \mathbf{0}$, and the crosscorrelation vector $\mathbf{r}(0) = \mathbf{0}$. The parameters λ is equal to 0.99. The results are obtained by averaging 100 independent runs. Both correlated input and white input are used. The correlated input of the system [7] has the following form

$$x(n) = 0.8x(n-1) + \beta(n), \tag{39}$$

where $\beta(n)$ was zero-mean white Gaussian noise with unit variance.

Figure 2 and Figure 3 show the MSE performance of the modified PU EDS for a time-invariant system with white input, and for PU length M=8 and M=4, respectively. We can see that all the PU EDS algorithms can converge the same steady-state MSE as the full update EDS for M=8. The MMax EDS has a converge rate close to the full update EDS. The sequential and stochastic methods have slightly higher steady-state MSE than the full update EDS when the PU length is M=4. The convergence rate of PU length M=4 is slower than that of PU length M=8.

Figure 4 and Figure 5 show the MSE performance of the modified PU EDS for a time-invariant system with correlated input vector $\mathbf{x}$, and for PU length M=8 and M=4, respectively. We can see that the MMax EDS still has the best performance among the different PU EDS algorithms. It can converge the same steady state MSE as the full update EDS. The convergence rate is also close to the full update EDS when M=8. The PU EDS are not stable with PU length M=4.

Table 1 shows the simulated MSE and theoretical MSE of PU EDS algorithms at steady state for white input. The

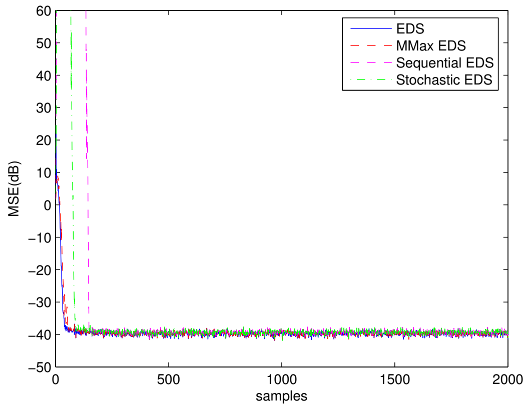

Fig. 2. Comparison of MSE of modified PU EDS with white input, M=8.

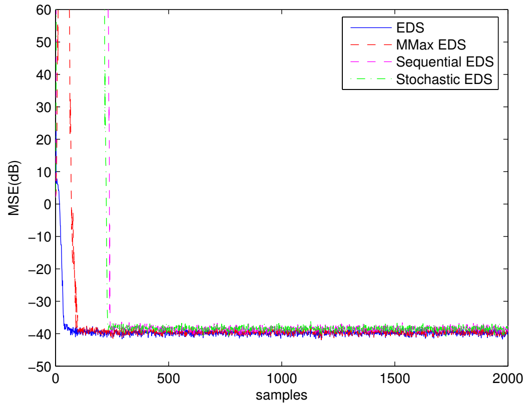

Fig. 3. Comparison of MSE of modified PU EDS with white input, M=4.

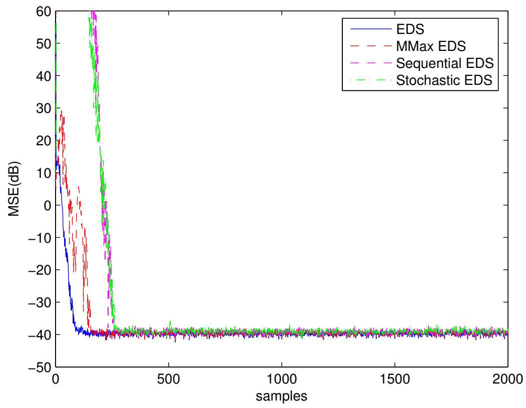

Fig. 4. Comparison of MSE of modified PU EDS with correlated input, M=8.

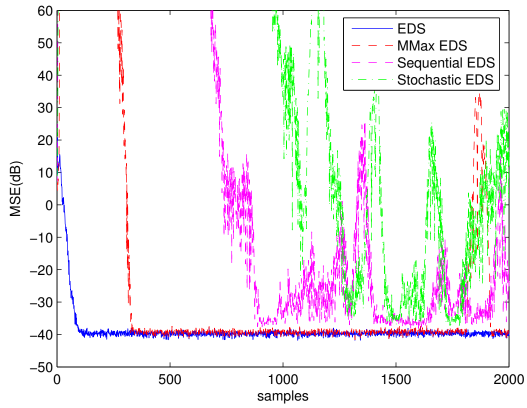

Fig. 5. Comparison of MSE of modified PU EDS with correlated input, M=4.

simulated results are obtained by taking the time average over the last 1000 samples. The theoretical results are calculated from (26). For full update EDS, $\kappa = 1$. We can see that the simulated results match the theoretical results.

Table 1. The simulated MSE and theoretical MSE of PU EDS for time-invariant system and white input.

Algorithms	Simulated MSE (dB)	Theoretical MSE (dB)
EDS (N=16)	-39.6617	-39.6641
MMax EDS (N=8)	-39.6247	-39.6056
Sequential EDS (N=8)	-39.2843	-38.8066
Stochastic EDS (N=8)	-39.2953	-38.8116
MMax EDS (N=4)	-39.5002	-39.3159
Sequential EDS (N=4)	-38.3864	-36.3638
Stochastic EDS (N=4)	-38.3792	-36.3640

5.2. Tracking performance of the PU EDS using the first-order Markov model

The same system identification model for a time-invariant system is used, except the weights are time-varying. The first-order Markov model (28) is used for the time-varying impulse response. The initial state of the impulse response is (38). The parameter γ in the first Markov model is 0.9998. The white process noise is used with difference variances. The white input signal with unity variance is used.

Figure 6 and Figure 7 show the tracking performance of the modified PU EDS with a different process noise $\sigma_\eta = 0.001$ and $\sigma_\eta = 0.01$ for $M = 8$. All PU EDS have similar performance. We can see that the MSE of PU EDS increases when the process noise increases. The variance of the MSE also increases when the process noise increases. The same situation also happens to the full-update EDS. Figure 8 and

Figure 9 show the tracking performance of the modified PU EDS with a different process noise $\sigma_\eta = 0.001$ and $\sigma_\eta = 0.01$ for $M = 4$. The partial update length does not have much effect on the MSE results in this case. The partial update length only affects the convergence rate. The convergence rate decreases as the partial update length decreases.

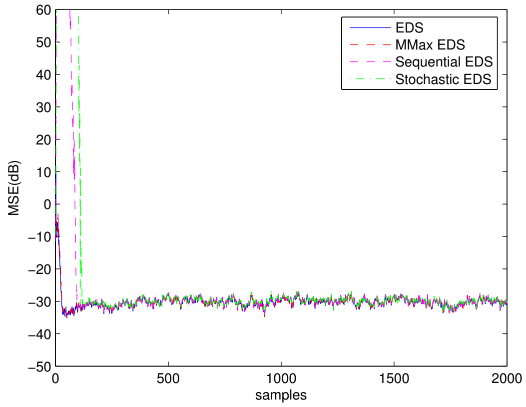

Fig. 6. Comparison of MSE of PU EDS with EDS for white input, N=16, M=8, $\sigma_\eta = 0.001$.

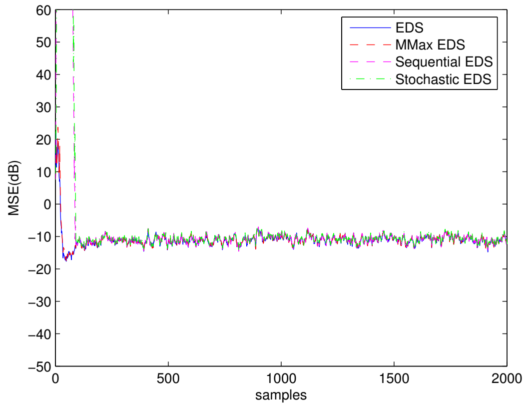

Fig. 7. Comparison of MSE of PU EDS with EDS for white input, N=16, M=8, $\sigma_\eta = 0.01$.

Table 2 and Table 3 show the simulated MSE and theoretical MSE of PU EDS algorithms at steady state for white input for process noise $\sigma_\eta = 0.001$ and $\sigma_\eta = 0.01$, respectively. The simulated results are obtained by taking the time average over the last 1000 samples. The theoretical results are calculated from (37). The partial-update lengths are $M = 8$ and $M = 4$. We can see that the simulated results match the theoretical results.

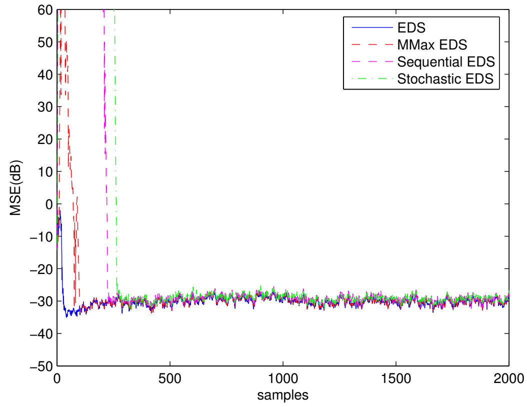

Fig. 8. Comparison of MSE of PU EDS with EDS for white input, N=16, M=4, $\sigma_\eta = 0.001$.

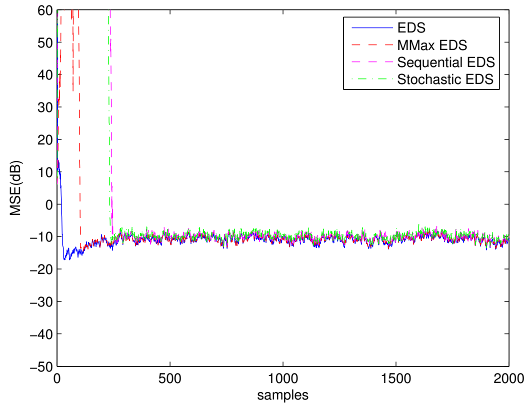

Fig. 9. Comparison of MSE of PU EDS with EDS for white input, N=16, M=4, $\sigma_\eta = 0.01$.

5.3. Tracking performance comparison of the MMax EDS with the EDS, RLS, MMax RLS, CG, and MMax CG

The tracking performance of the MMax EDS is also compared with the full-update EDS, full-update RLS, MMax RLS, full-update CG, and MMax CG. The same system identification model is used. After 2000 samples/iterations pass, the unknown system in (38) is changed by multiplying all coefficients by -1. Figure 10 shows the MSE results among EDS, MMax EDS, RLS, MMax RLS, CG, and MMax CG when $M = 4$. White input is used. The results show that these algorithms have a similar convergence rate after the unknown system is changed. The EDS and MMax EDS have convergence rates very close to the full update RLS. The SORTLINE sorting method is used for MMax EDS, MMax RLS, and MMax CG. The total number of multiplications of MMax EDS, MMax RLS, MMax CG are

Table 2. The simulated MSE and theoretical MSE of PU EDS for process noise $\sigma_\eta = 0.001$.

Algorithms	Simulated MSE (dB)	Theoretical MSE (dB)
EDS (N=16)	-29.9657	-30.4766
MMax EDS (M=8)	-29.9571	-30.4699
Sequential EDS (M=8)	-29.7107	-30.3654
Stochastic EDS (M=8)	-29.6381	-30.3650
MMax EDS (M=4)	-30.2689	-30.4349
Sequential EDS (M=4)	-29.3666	-29.9289
Stochastic EDS (M=4)	-29.2707	-29.9384

Table 3. The simulated MSE and theoretical MSE of PU EDS for process noise $\sigma_\eta = 0.001$.

Algorithms	Simulated MSE (dB)	Theoretical MSE (dB)
EDS (N=16)	-10.7191	-11.0287
MMax EDS (M=8)	-10.6829	-11.0286
Sequential EDS (M=8)	-10.4600	-11.0273
Stochastic EDS (M=8)	-10.4281	-11.0274
MMax EDS (M=4)	-10.5673	-11.0282
Sequential EDS (M=4)	-9.7506	-11.0221
Stochastic EDS (M=4)	-9.6415	-11.0222

$N^2 + 2NM + N + 2M$, $2N^2 + 2NM + 3N + M + 1$, and $2N^2 + M^2 + 9N + M + 3$, respectively. In this case, the full update length N is 16. The partial update length M is 4. The detailed computational complexities of these algorithms are shown in Table 4. Overall, the EDS algorithms need fewer multiplications than the RLS and CG. The results show that the MMax EDS with M=4 can achieve similar tracking performance to the full-update EDS and RLS while reducing the computational complexity significantly.

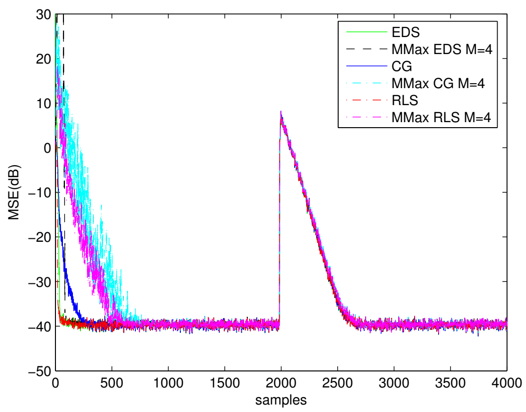

Fig. 10. Comparison of MSE of MMax EDS with EDS, RLS, MMax RLS, CG, and MMax CG for white input, N=16, M=4.

Table 4. The computational complexities of EDS, MMax EDS, RLS, MMax RLS, CG, and MMax CG.

Algorithms	Number of multiplications per symbol	Number of comparisons per symbol
EDS (N=16)	816	–
MMax EDS (M=4)	408	10
RLS (N=16)	3721	–
MMax RLS (M=4)	693	10
CG (N=16)	3003	–
MMax CG (M=4)	679	10

6. CONCLUSION

In this paper, the PU EDS is modified to achieve better performance. The performance is analyzed for a time-invariant system and for a time-varying system. Theoretical steady-state mean and MSE results of the modified PU EDS are derived for both the time-invariant system and time-varying system. Simulation results agree with the derived theoretical results in steady state. The PU EDS can reduce the computational complexity significantly. The PU EDS can achieve comparable performance to the full update EDS when the convergence condition is satisfied. The tracking performance of the MMax EDS is also compared with the full update EDS, full update RLS, MMax RLS, full update CG, and MMax CG. The results show that the MMax EDS can achieve similar tracking performance to the full update EDS and full update RLS, while reducing the computational complexity significantly.

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