Modified Partial Update EDS Algorithms for Adaptive Filtering

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Motivation

- EDS algorithm has high computational complexity $O(N^2)$.
- Partial update methods have been applied to reduce the computational complexity of EDS, but the performance was not good.
- What is the performance of modified partial update EDS?

Euclidean Direction Search Algorithm

Solve system with form $\mathbf{Q}\mathbf{w} = \mathbf{r}$

A direction search method is used for minimizing the cost function

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \alpha \mathbf{g}(n)$$

g(n) is the direction vector

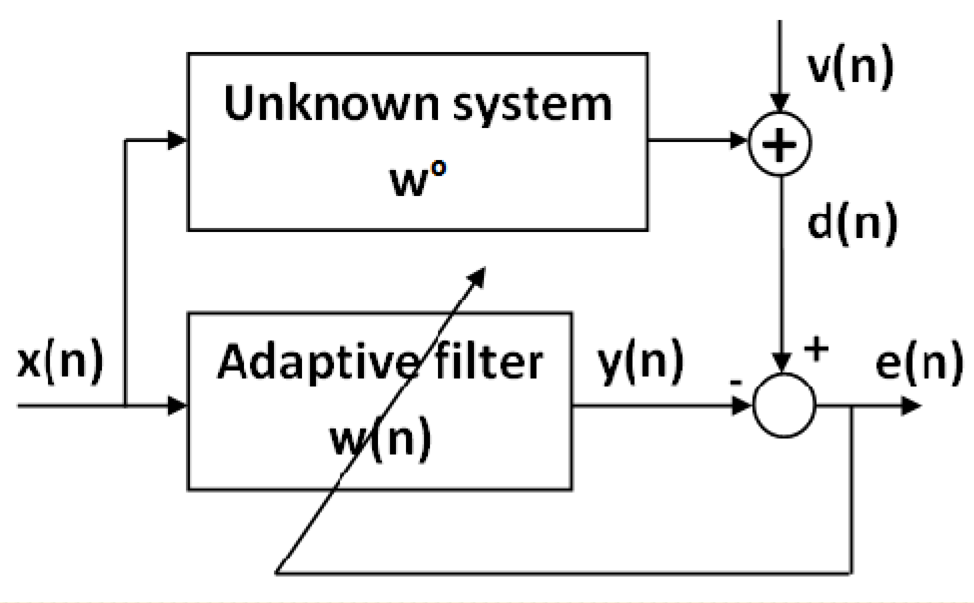
Euclidean direction is used

$$\mathbf{g}_i = [0, \cdots, 0, 1, 0, \cdots 0]^T$$

The 1 appears in the *i*-th position

EDS in Adaptive Filter System

A system identification model is



$$d(n) = \mathbf{x}^T(n)\mathbf{w}^o + v(n)$$

To estimate the $\mathbf{Q}$ and $\mathbf{r}$ in $\mathbf{Qw}=\mathbf{r}$, the exponentially decaying data window is used

$$\mathbf{Q}(n)=\sum_{i=0}^{n}\lambda^{n-i}\mathbf{x}(i)\mathbf{x}^{T}(i)=\lambda\mathbf{Q}(n-1)+\mathbf{x}(n)\mathbf{x}^{T}(n)$$

$$\mathbf{r}(n)=\sum_{i=0}^{n}\lambda^{n-i}d(i)\mathbf{x}(i)=\lambda\mathbf{r}(n-1)+d(n)\mathbf{x}(n)$$

λ is the forgetting factor

$$\mathbf{w}(n+1)=\mathbf{w}(n)-\frac{\mathbf{g}\mathbf{g}^{T}\left(\mathbf{Q}(n)\mathbf{w}(n)-\mathbf{r}(n)\right)}{\mathbf{g}^{T}\mathbf{Q}(n)\mathbf{g}}$$

Partial Update (PU) Methods

- Update part of the weights to save the computational complexity
- Each update step, update M<N coefficients
- Basic PU methods include periodic, sequential, stochastic, and MMax methods
  - The periodic method: update the weights at every S^{th} iteration and copy the weights at the other iterations, where $S = \lceil N/M \rceil$

- The sequential method: choose the subset of the weights in a round-robin fashion.
- The stochastic method: is a randomized version of the sequential method. Usually a uniformly distributed random process will be applied.
- The MMax method: the elements of the weight **w** are updated according to the position of the M largest elements of the input vector **x**(n).

Modified Partial Update EDS Algorithm

The modified partial update EDS algorithm has the uniform update equation

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mathbf{g}\mathbf{g}^T\left(\tilde{\mathbf{Q}}(n)\mathbf{w}(n) - \hat{\mathbf{r}}(n)\right)}{\mathbf{g}^T\tilde{\mathbf{Q}}(n)\mathbf{g}}$$

$$\tilde{\mathbf{Q}}(n) = \lambda\tilde{\mathbf{Q}}(n-1) + \mathbf{x}(n)\hat{\mathbf{x}}^T(n)$$

$$\hat{\mathbf{r}}(n) = \lambda\hat{\mathbf{r}}(n-1) + d(n)\hat{\mathbf{x}}(n)$$

Old PU EDS used $\hat{\mathbf{Q}}(n) = \lambda\hat{\mathbf{Q}}(n-1) + \hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)$

$$\hat{\mathbf{x}}(n) = \mathbf{I}_M(n)\mathbf{x}(n)$$

$$\mathbf{I}_M(n) = \begin{bmatrix} i_1(n) & 0 & \cdots & 0 \\ 0 & i_2(n) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & i_N(n) \end{bmatrix}$$

$$\sum_{k=1}^{N} i_k(n) = M, \quad i_k(n) \in \{0,1\}$$

The number of multiplications of EDS is $3N^2+3N$ per sample

The number of multiplications of modified PU EDS is $N^2+2NM+N+2M$ per sample

Steady-state Performance Analysis in a Time-invariant System

Assumptions;

- Inverse of $\tilde{\mathbf{Q}}(n)$ exists
- The coefficient error w(n)- $w^o(n)$ is small and independent of the input signal x(n) at steady state
- White noise v(n) and input signal x(n) are independent

At steady state, the MSE of PU EDS with correlated input is

$$E\left\{ \left|e(n)\right|^2 \right\} \approx \sigma_v^2 + tr\left(\mathbf{Q}\left(\frac{1-\lambda}{1+\lambda}\sigma_v^2 \tilde{\mathbf{Q}}^{-1}\hat{\mathbf{Q}}\tilde{\mathbf{Q}}^{-T} \right)\right)$$

$$\hat{\mathbf{Q}} = E\left\{\hat{x}(n)\hat{x}^T(n)\right\}$$

$$\sigma_v^2 = E\left\{v^2(n)\right\}$$ Variance of noise

At steady state, the MSE of PU EDS with white input is

$$E\left\{ \left|e(n)\right|^2 \right\} \approx \sigma_v^2 + \frac{N(1-\lambda)}{1+\lambda} \sigma_v^2 \sigma_x^2 \sigma_{\hat{x}}^2 \sigma_{\tilde{x}}^{-4}$$

$$\sigma_x^2 = tr(\mathbf{Q})$$

$$\sigma_{\hat{x}}^2 = tr(\hat{\mathbf{Q}})$$

$$\sigma_{\tilde{x}}^2 = tr(\tilde{\mathbf{Q}})$$

$$\sigma_v^2 = E\left\{v^2(n)\right\}$$ Variance of noise

Tracking Performance Analysis

Desired signal becomes:

$$d(n) = \mathbf{x}^T(n)\mathbf{w}^o(n) + v(n)$$

Time-varying system $\mathbf{w}^o(n)$ uses a first-order Markov model

$$\mathbf{w}^o(n) = \gamma \mathbf{w}^o(n-1) + \boldsymbol{\eta}(n)$$

γ is very close to unity

$\boldsymbol{\eta}(n)$ is process noise

Additional assumption:

- White noise v(n), input signal **x**(n), and process noise **η**(n) are independent of each other

At steady state, the MSE of PU EDS with correlated input is

$$E\left\{ \left|e(n)\right|^2 \right\} \approx \sigma_v^2 + tr\left(\mathbf{Q}\left(\frac{1-\lambda}{1+\lambda}\sigma_v^2 \tilde{\mathbf{Q}}^{-1}\hat{\mathbf{Q}}\tilde{\mathbf{Q}}^{-T} \right)\right)$$

$$+ \frac{\lambda^2}{1-\lambda^2}\sigma_x^2 tr(\mathbf{Q}_\eta)$$

$$\hat{\mathbf{Q}} = E\left\{\boldsymbol{\eta}(n)\boldsymbol{\eta}^T(n)\right\}$$

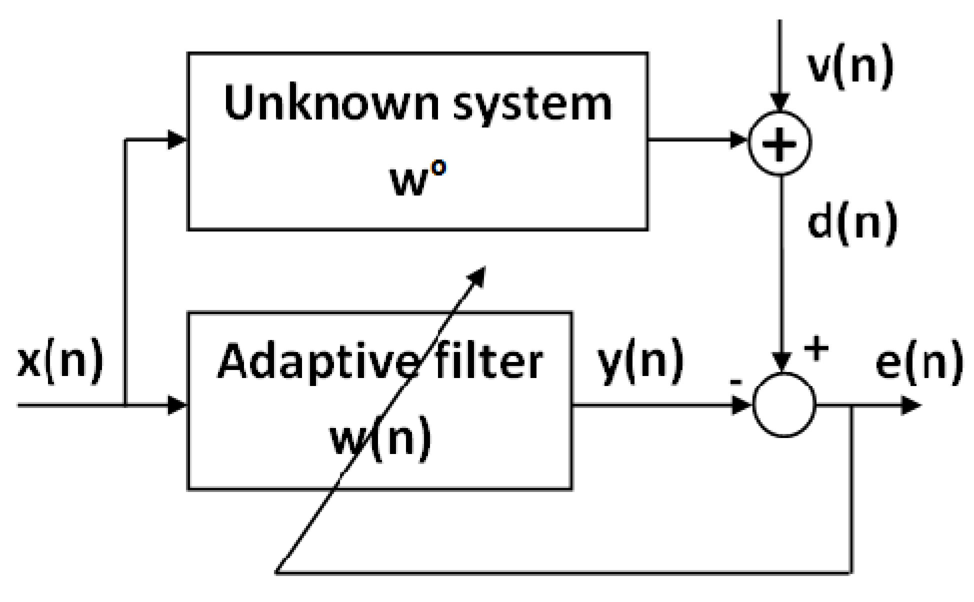
At steady state, the MSE of PU EDS with white input is

$$E\left\{ \left|e(n)\right|^2 \right\} \approx \sigma_v^2 + \frac{N(1-\lambda)}{1+\lambda}\sigma_v^2\sigma_x^2\sigma_{\hat{x}}^2\sigma_{\tilde{x}}^{-4}$$

$$+ \frac{\lambda^2}{1-\lambda^2}\sigma_x^2 tr(\mathbf{Q}_\eta)$$

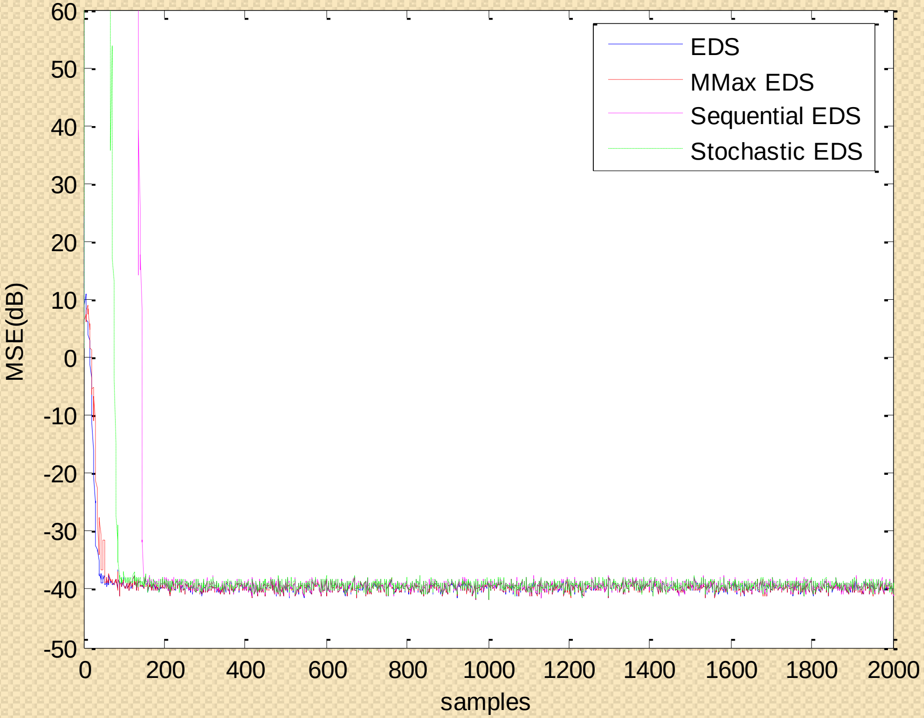
Simulations

System identification model

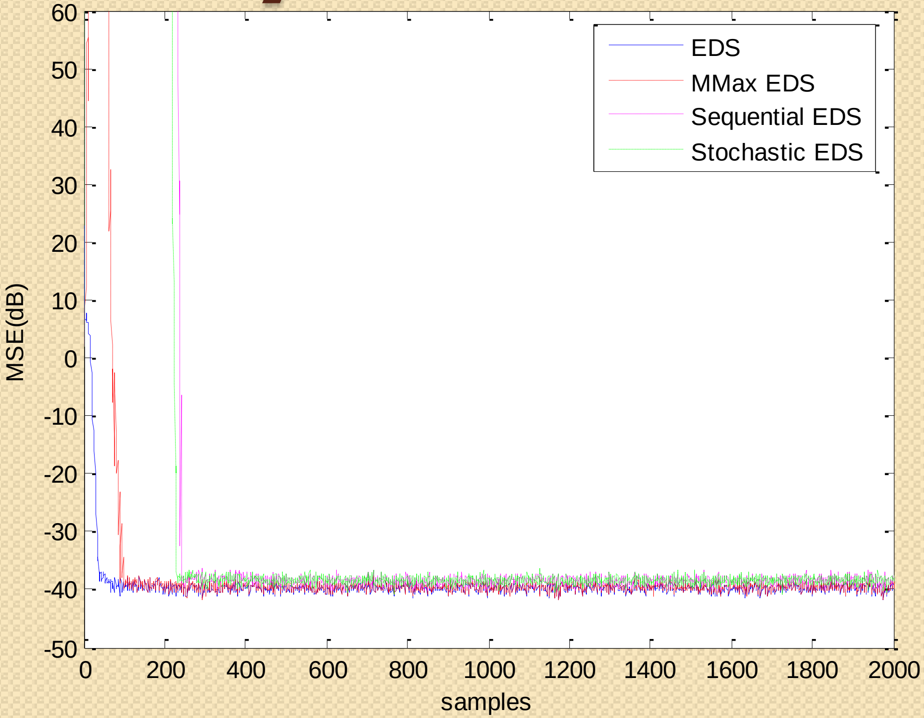


The initial impulse response of unknown system is 16-order (N=16) FIR filter
$\mathbf{w}^{o}$=[0.01,0.02,-0.04,-0.08,0.15,-0.3,0.45, 0.6, 0.6,0.45,-0.3,0.15,-0.08,-0.04,0.02,0.01]^{T}

Time-invariant System



M = 8



M = 4

The variance of the input noise σ_v^2=0.0001

White input, variance is 1

λ = 0.99

Table 1. The simulated MSE and theoretical MSE of PU EDS for time-invariant system and white input.

Algorithms	Simulated MSE (dB)	Theoretical MSE (dB)
EDS (N=16)	-39.6617	-39.6641
MMax EDS (N=8)	-39.6247	-39.6056
Sequential EDS (N=8)	-39.2843	-38.8066
Stochastic EDS (N=8)	-39.2953	-38.8116
MMax EDS (N=4)	-39.5002	-39.3159
Sequential EDS (N=4)	-38.3864	-36.3638
Stochastic EDS (N=4)	-38.3792	-36.3640

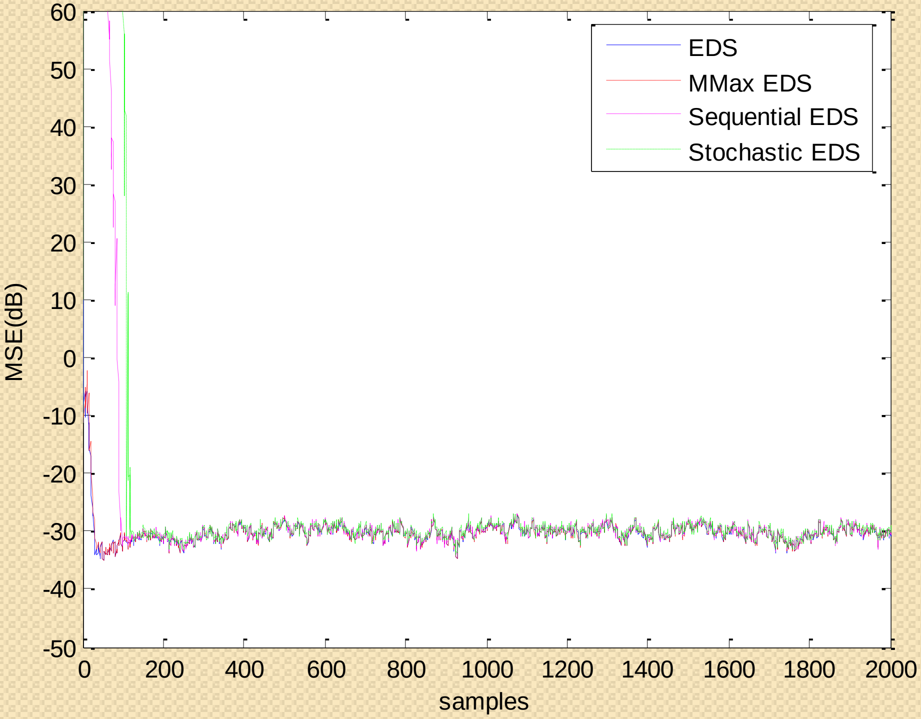
Tracking Performance

The variance of the input noise $\sigma_v^2=0.0001$

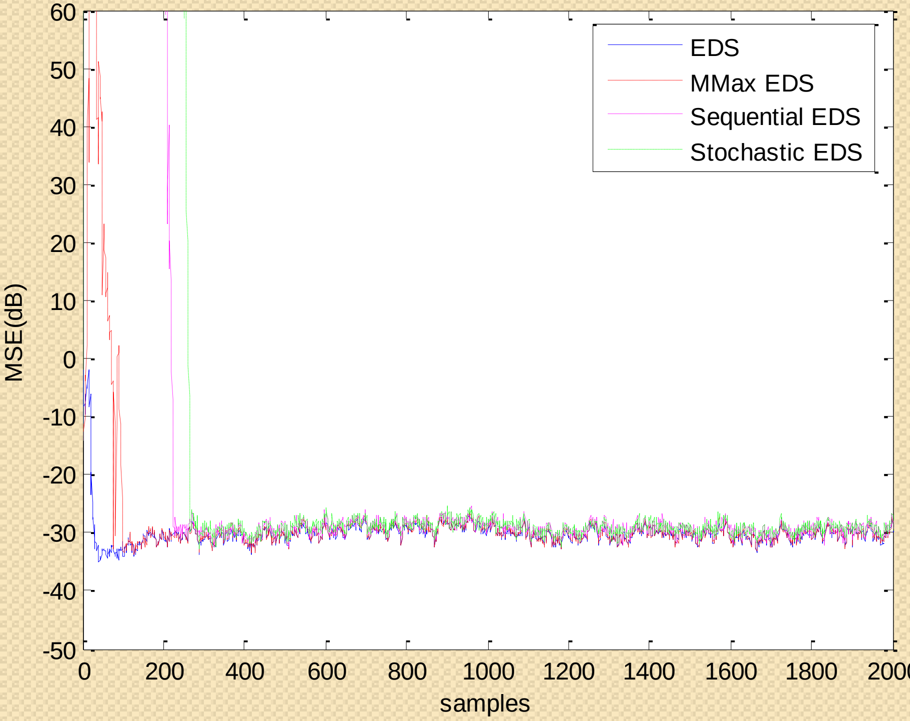
White input, variance is 1

$\lambda = 0.99$

γ in Markov model is 0.9998

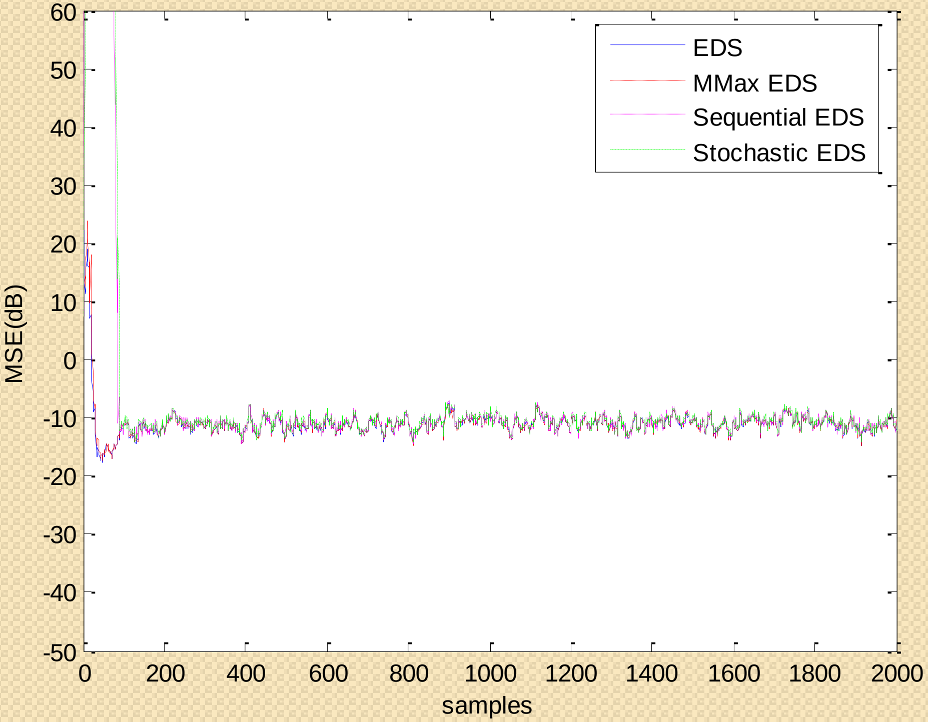


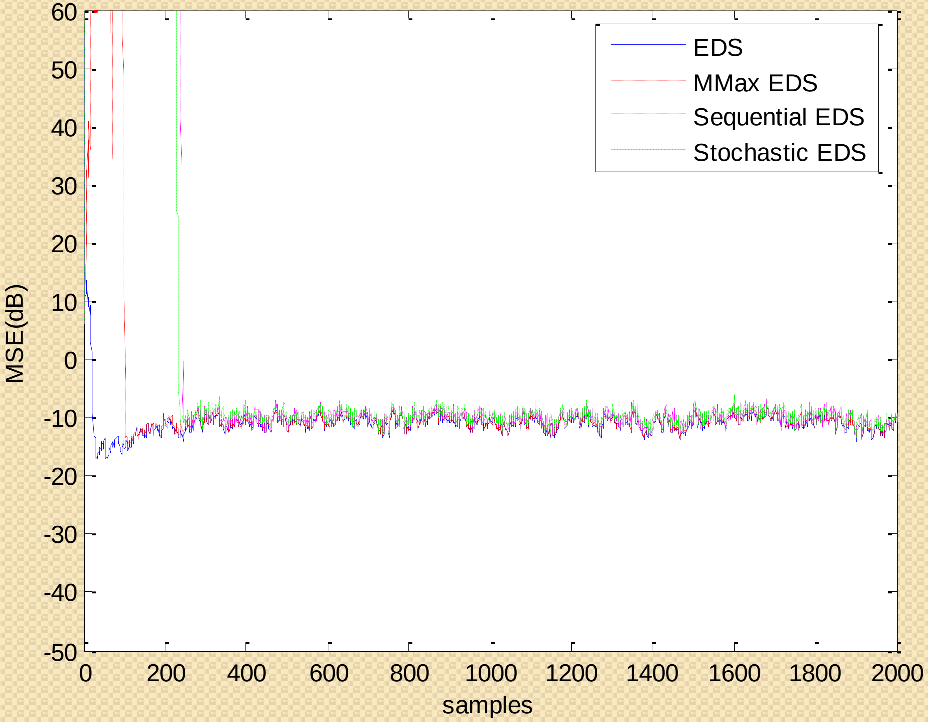
M = 8



M = 4

process noise $\sigma_\eta = 0.001$





M = 8

M = 4

process noise $\sigma_\eta = 0.01$

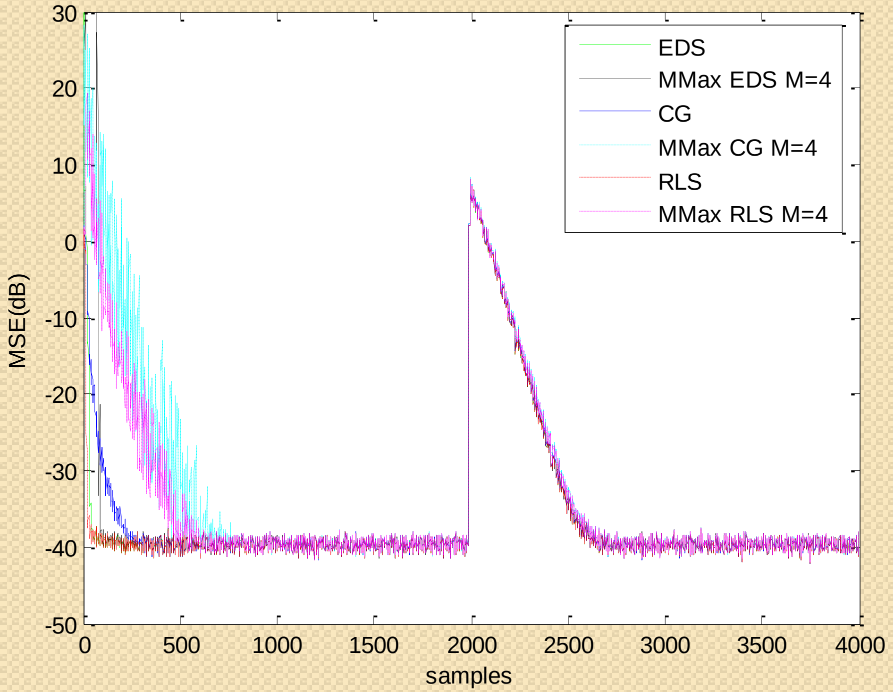
Table 2. The simulated MSE and theoretical MSE of PU EDS for process noise = 0.001.

Algorithms	Simulated MSE (dB)	Theoretical MSE (dB)
EDS (N=16)	-29.9657	-30.4766
MMax EDS (N=8)	-29.9571	-30.4699
Sequential EDS (N=8)	-29.7107	-30.3654
Stochastic EDS (N=8)	-29.6381	-30.3650
MMax EDS (N=4)	-30.2689	-30.4349
Sequential EDS (N=4)	-29.3666	-29.9289
Stochastic EDS (N=4)	-29.2707	-29.9384

Table 3. The simulated MSE and theoretical MSE of PU EDS for process noise = 0.01.

Algorithms	Simulated MSE (dB)	Theoretical MSE (dB)
EDS (N=16)	-10.7191	-11.0287
MMax EDS (N=8)	-10.6829	-11.0286
Sequential EDS (N=8)	-10.4600	-11.0273
Stochastic EDS (N=8)	-10.4281	-11.0274
MMax EDS (N=4)	-10.5673	-11.0282
Sequential EDS (N=4)	-9.7506	-11.0221
Stochastic EDS (N=4)	-9.6415	-11.0222

Tracking Performance comparison among MMax EDS, Mmax CG, and MMax RLS



Comparison of MSE of MMax EDS with EDS, CG, MMax CG, RLS, and MMax RLS for white input, N=16, M=4.

After 2000 samples/iterations pass, the unknown system is changed by multiplying all coefficients by -1.

The partial update length only affects the convergence rate at the beginning in this case.

Table 4. The computational complexities of EDS, MMax EDS, RLS, MMax RLS, CG, and MMax CG.

Algorithms	Number of multiplications per symbol	Number of comparisons per symbol
EDS (N=16)	816	--
MMax EDS (M=4)	408	10
RLS (N=16)	3721	--
MMax RLS (M=4)	693	10
CG (N=16)	3003	--
MMax CG (M=4)	679	10

Summary

- The PU EDS is modified to achieve better steady-state performance
- The performance is analyzed for a time-invariant system and for a time-varying system
- The PU EDS can achieve comparable performance to the full update EDS
- The PU EDS can reduce the computational complexity significantly
- Comparing with MMax RLS and MMax CG, the MMax EDS algorithm has lowest computational complexity while achieving the similar tracking performance