FREQUENCY DOMAIN EYE DIAGRAM FOR ORTHOGONAL FREQUENCY-DIVISION MULTIPLEXING

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***Abstract*—In wireless communications systems, information symbols are represented in various dimensions of electro space such as time, frequency, and code. Impairments in transmit channel have different effects on the symbols in dimensions. Visualization and measurement of distortions in various domains need to be addressed for better understanding of multidimensional communication systems and their performance under distortion. Therefore, in this paper, eye diagram in frequency domain for orthogonal frequency-division multiplexing (OFDM) based signals is introduced for visualization and identification of time varying impairments like frequency spread, frequency offset, and phase noise. Interpolation between output samples of discrete Fourier transform (DFT) is performed by zero padding into received time domain symbols. Proposed method provides identity information about time varying impairments as well as quantity information about frequency offset. Real channel measurements are performed to illustrate the effectiveness of the method in practical cases. The frequency domain eye diagram can be employed to test the radio equipment and wireless communication systems, and for educational purposes.**

***Index Terms*—Frequency domain eye diagram, interference identification, OFDM.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) is adopted by wireless communication systems recently because of its ability to effectively convert frequency selective channel to multiple flat subchannels that can easily be handled by one tap frequency domain equalization [1], adaptive and flexible bandwidth utilization [2], and higher spectral efficiency with overlapping subcarriers. Despite the advantages in time dispersive channels, high spectral sidelobes of overlapping subcarriers make OFDM very sensitive to inter-carrier interference (ICI) in the presence of time varying impairments like Doppler spread, carrier frequency offset (CFO), and phase noise.

Baseband symbols in wireless communications are defined in various dimensions in which transmit channel has different effects. Identification of impairment source is critical for complete system analysis. Therefore, in testing and measurements, channel impact on signals in different dimensions should be considered. For example, eye diagrams have been heavily used to observe the impairments in single carrier systems like Global System for Mobile Communications (GSM) in which the symbols are defined in time domain. However in OFDM based systems, transmitted symbols are modulated along subcarriers that are defined in frequency domain. Therefore, an eye diagram in frequency domain for OFDM subcarriers can be used as a tool to achieve the detection and visualization of the time varying channel impairments while inter-symbol interference caused by channel impairments like delay spread and timing jitter for single carrier signals is observed by time domain eye diagram.

Table I: Impairments by domain and source. Frequency domain eye diagram is to observe time varying impairments

		Domain	
		Time	Frequency
Source	Channel	Delay spread	Doppler Spread
	Hardware	Timing jitter	Frequency offset
Visualization		Time Domain Eye Diagram	Frequency Domain Eye Diagram

Use of eye diagrams for communication signals in time domain is extensively discussed in [3] and eye pattern is introduced for noncoherent receivers including estimation of required carrier to noise ratio for required error rate in [4]. Time domain eye diagram is a powerful tool to investigate interference between consecutive time domain symbols. Overlapping of OFDM subcarriers in frequency is analogous to time overlapping of Nyquist pulses in single carrier communications. For example, raised cosine filter is the most popular pulse shaping filter that satisfies the Nyquist criterion [5]. However, zero-inter-symbol interference (ISI) property of Nyquist pulse shaped single carrier signals is distorted in multipath channels. Timing jitter at the receiver also causes ISI. Similar to impact of channel impairments like delay spread and timing jitter on zero-ISI property of Nyquist filtered single carrier signals, time varying channels disturb the zero-ICI property of the OFDM subcarriers.

Inspired by this analogy, frequency domain eye diagram (FDED) is introduced in order to investigate the distortions like Doppler Spread and CFO as summarized in Table I. The use of FDED in testing and validation of the software defined radio systems provides useful information whether carrier frequency offset and frequency spreading occur or not by considering shift of eye crossings and reduced eye openings in the analysis, respectively. Frequency offset estimation without utilizing preambles is also possible by determining

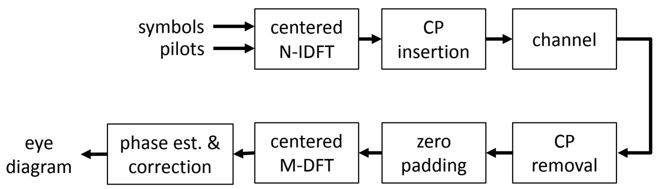


Fig. 1: Block diagram for FDED

the shift of eye pattern that minimizes the error magnitude of interpolated points. Also, as a visualization tool, the technique can be implemented for educational purposes.

This paper is organized as follows: In Section II, system model is introduced. Section III demonstrates signal generation and construction of FDED with examples. Measurement results for various channel scenarios are given in Section IV. Finally, conclusions are drawn in section V.

II. System Model

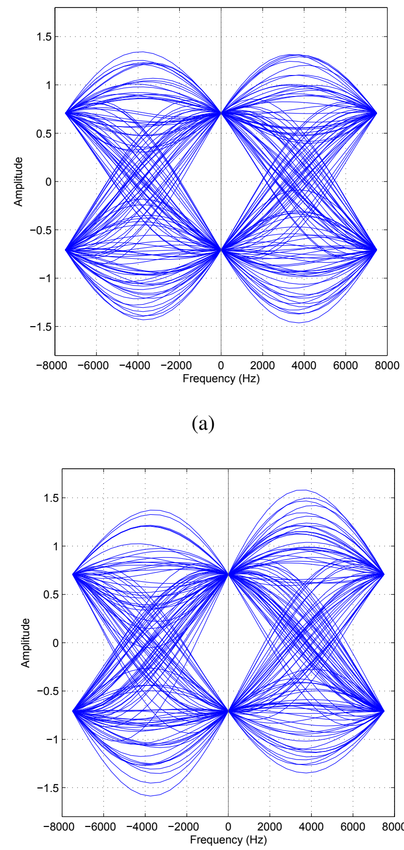
We consider an OFDM system in which transmitter generates symbols by processing complex baseband symbol vector with centered inverse discrete Fourier transform (IDFT) operation. Since the time indexes of regular IDFT and discrete Fourier transform (DFT) operations are defined from zero to $N-1$, zero padding in time distorts the symmetric property of transform pairs which results in complex interpolation between DFT points. Rotation of interpolated samples prevents to obtain clear eye patterns for OFDM subcarriers. In order to avoid the complex interpolation, centered DFT is implemented for OFDM signal generation and reception, that also provides full analogy with continuous Fourier transform [6]. After the addition of cyclic prefix (CP), the OFDM symbol is converted to an analog time-domain signal before being transmitted through channel. The sampled time-domain baseband representation for one OFDM symbol at the transmitter becomes

$$x(n) = \sum_{k=-\frac{N-1}{2}}^{\frac{N-1}{2}} X(k) e^{j\frac{2\pi}{N}kn}, \quad n = -N_{\text{CP}} - \frac{N-1}{2}, \ldots, \frac{N-1}{2} \tag{1}$$

where n is sampled time index, $X(k)$ is complex data symbol assigned to kth subcarrier, N is IDFT size which is also number of subcarriers, N_{CP} is CP size in DFT samples and j is imaginary unit.

III. Frequency Domain Eye Diagram

Eye diagram is constructed by having denser interpolation between DFT output samples at the receiver. By zero padding to the sampled received signal and performing greater size centered DFT, analysis operation is made closer to discrete-time Fourier transform (DTFT). Fig.1 shows block diagram for OFDM system with eye diagram construction.



(a)

(b)

Fig. 2: Frequency domain eye diagrams for (a) in-phase and (b) quadrature components

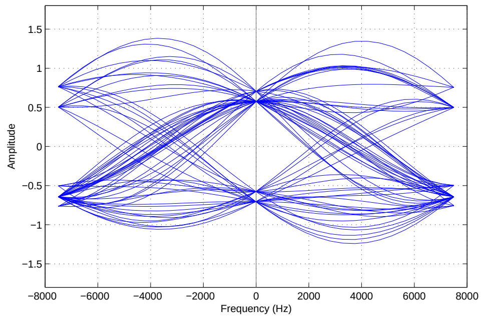
At the receiver end, CP is removed from the received OFDM symbols after passing through the channel. Then, zero padding and centered DFT are performed in the reception stage as follows

$$X_{ip}(l) = \sum_{m=-\frac{M-1}{2}}^{\frac{M-1}{2}} x_{zp}(m) e^{-j\frac{2\pi}{M}lm}, \quad l = -\frac{M-1}{2}, \ldots, \frac{M-1}{2} \tag{2}$$

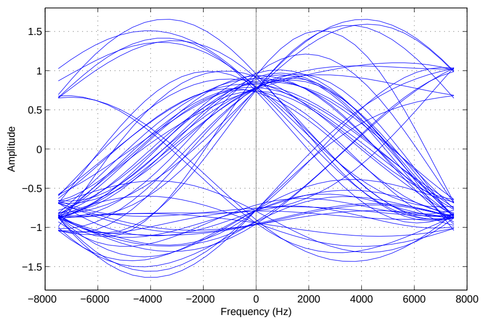
where l is index of interpolated frequency components of received OFDM signal, $M \geq N$ is DFT size and m is sampled time index of the zero padded signal

$$x_{zp}(m) = \begin{cases} 0 & -\frac{M-1}{2} \leq m < -\frac{N-1}{2} \\ x(m) & -\frac{N-1}{2} \leq m \leq \frac{N-1}{2} \\ 0 & \frac{N-1}{2} < m \leq \frac{M-1}{2}. \end{cases} \tag{3}$$

Interpolated frequency domain OFDM symbols are used to construct eye patterns. As the greater DFT size is achieved at the receiver, eye diagram for both in phase and quadrature



(a)



(b)

Fig. 3: Eye diagrams of 4 subcarriers and N/4 symbols (a) without and (b) with Doppler effect in multipath channel.

part of complex baseband symbols are constructed via partitioning the interpolated frequency domain symbols into two carrier spacing-sized chunks. Then, each chunk is plotted at the top of each other to construct the eye diagrams. At the final step, two diagrams are obtained for each symbol: Real part of complex subcarriers, $\Re\{X_{ip}(l)\}$, is for eye diagram of in-phase component of the symbols which are modulated on subcarriers while imaginary part, $\Im\{X_{ip}(l)\}$, is for eye diagram of quadrature component. Fig.2 shows simulated diagrams for in-phase and quadrature parts of quadrature phase-shift keying (QPSK) symbols without channel effect, with $N = 256$, $M = 4096$ and 7.5 kHz subcarrier spacing.

FDED is considered for time-varying impairments; however, plotting N subcarriers on top of each other for each OFDM symbol distorts eye openings in frequency selective channels. In order to mitigate the effect of frequency selectivity, number of subcarriers is reduced while consecutive OFDM symbols are included in diagram construction. Fig.3 shows simulation results for eye diagram in ITU Pedestrian A multipath channel [7] with $N/4$ subcarriers of 4 symbols keeping number of total components the same. Two knobs at each sampling point of subcarriers are observed in Fig.3(a) because of different channel gains at frequencies corresponding to the those subcarriers. Fig.3(b) shows the effect of Doppler spread in the channel.

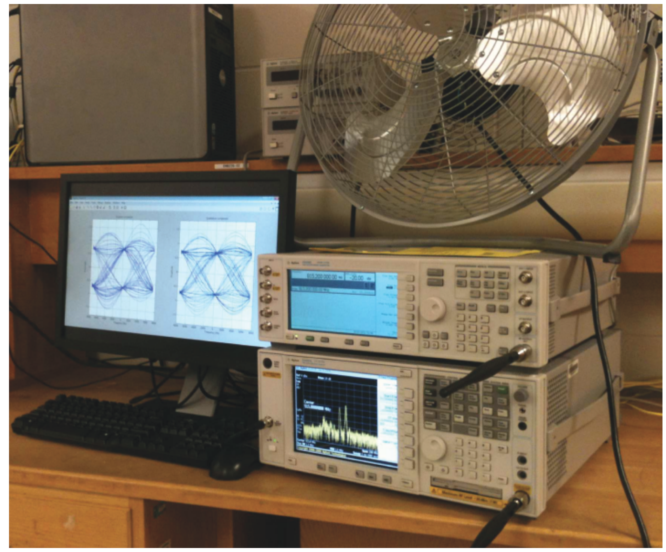
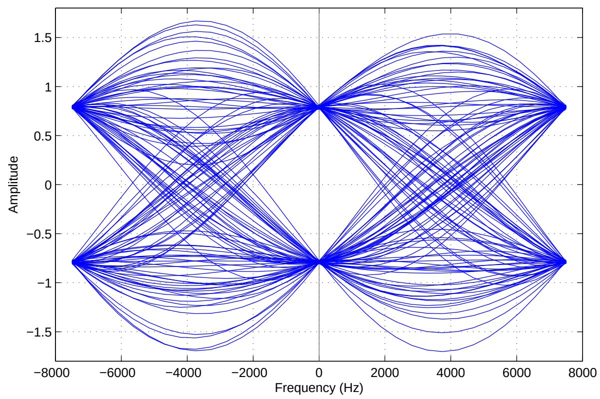


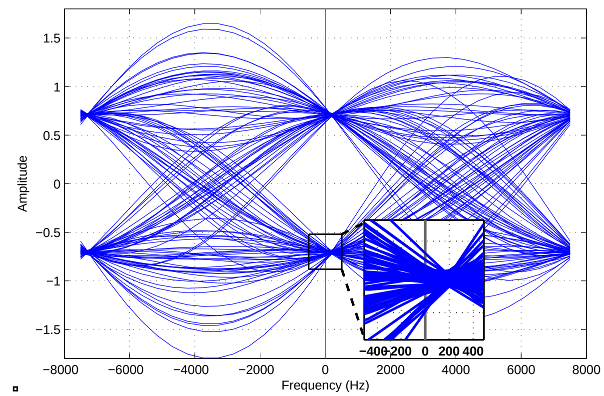
Fig. 4: Laboratory setup for eye diagram measurements

IV. Measurement Results

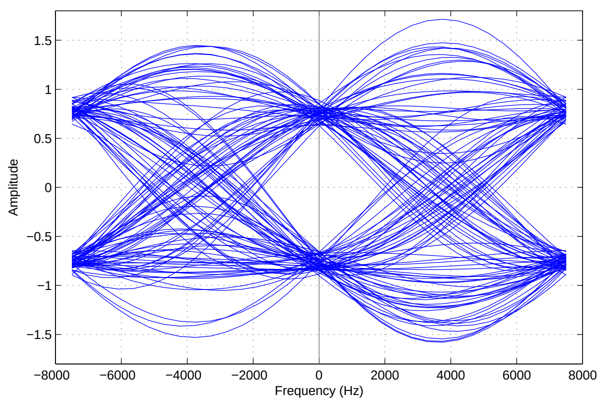
FDED is tested in wireless communication systems laboratory in order to confirm the in practical utility under real channel conditions. Baseband OFDM signal is generated in software and pushed to the Agilent E4438C ESG Vector Signal Generator. Transmitted signal is captured by Agilent E4440A PSA Spectrum Analyzer after passing through wireless channel. Received signal is downloaded to the computer and FDED is constructed in the software. Intentional CFO is inserted by manually changing carrier frequency of the spectrum analyzer and time variation is achieved by putting a rotating fan between transmitter and receiver antennas with a setup as shown in Fig.4. Subcarrier spacing and number of subcarriers are set to 7.5 kHz and 256 in the measurements. To have smooth diagram, interpolation ratio, M/N, is selected as 16 which corresponds to 15 additional eye pattern samples for each DFT sample. Eye diagram for in-phase component of QPSK modulated OFDM signal is given in Fig.5a, without impairment in the channel. Since the main consideration is time varying impairments, OFDM signal is tested in the presence of time variation in the channel, as well as CFO. For all measurements, common oscillator is used for transmitter and receiver in order not to have uncontrolled frequency offset between local oscillators. For frequency offset measurement, intentional 200 Hz CFO is created at the receiver. Introduced frequency offset is clearly observed as in Fig.5b. Frequency offset estimation can be performed by minimizing the error magnitude of interpolated subcarriers and with image processing tools, which is out of scope of this study. Time selective channel is carried out by placing a rotating 3-blade fan between transmitter and receiver antennas. Fig.5c shows the measured eye diagram when the fan is operating, i.e. creating a time varying



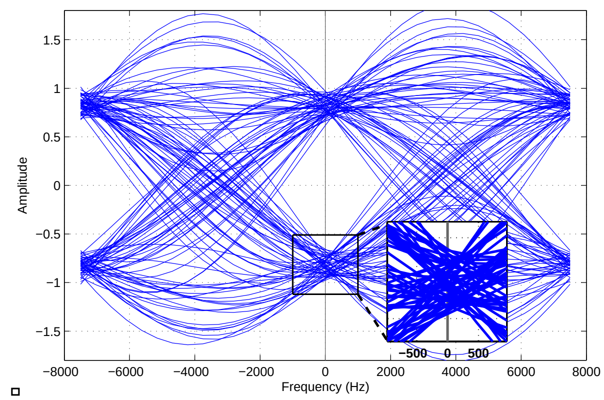
(a) Without intentional impairment



(b) With 200 Hz intentional CFO



(c) Rotating fan is ON



(d) With 200 Hz intentional CFO and fan is ON

Fig. 5: Eye diagram measurements for in-phase components (a) without and (b) with CFO. (c) Frequency spread is observed with rotating fan and (d) CFO and frequency spread occurs at the same time.

channel. Spread in the frequency is observed as distortion in the eye diagram in both directions along frequency. Presence of both CFO and Doppler spread is also investigated. When Doppler spread and CFO occur at the same time, distorted eye diagram shifts with the amount of frequency difference between transmitter and receiver as in Fig.5d, which also makes frequency offset estimation in time variation possible.

V. Conclusions

FDED has been defined for subcarriers of OFDM signals. As time domain eye diagram is used for understanding the impact of impairments on the single carrier waveforms that are transmitted in time domain, time varying impairments like frequency spread and CFO in multicarrier waveforms where the transmitted symbols are defined in frequency domain are visualized and identified from each other with eye patterns in frequency. Proposed method also provides frequency offset estimation without using preamble. FDEDs can be used for testing of OFDM systems, for conducting interference measurements and for educational purposes.

Acknowledgment

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References

[1] J. Cimini, L., "Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiplexing," *IEEE Trans. Commun.*, vol. 33, no. 7, pp. 665 – 675, Jul. 1985.

[2] T. Weiss and F. Jondral, "Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency," *Communications Magazine, IEEE*, vol. 42, no. 3, pp. S8 – 14, Mar. 2004.

[3] R. Lucky, J. Salz, and E. Weldon, *Principles of Data Communication*. McGraw-Hill, 1968.

[4] W. McGee, "Eye Pattern for the Binary Noncoherent Receiver," *IEEE Trans. Commun. Technol.*, vol. 19, no. 5, pp. 634 – 643, Oct. 1971.

[5] T. S. Rappaport, *Wireless Communications: Principles and Practice (2nd Edition)*. Prentice Hall, 2002.

[6] D. Mugler, "The Centered Discrete Fourier Transform and a parallel implementation of the FFT," in *2011 IEEE International Conf. Acoustics, Speech and Signal Processing (ICASSP)* , pp. 1725 –1728.

[7] "Rec. ITU-R M.1225: Guidelines for Evaluation of Radio Transmission Technologies for IMT-2000," International Telecommunication Union, 1997.